

DOK PROGRESSION GUIDE • SBAC MATHEMATICS

Math 2

Every SBAC claim and target for Math 2, with a Test Bank question at each Depth of Knowledge (DOK) level the blueprint assesses, and the clicks to find more.

DOK 1 Recall

DOK 2 Skills & concepts

DOK 3 Strategic thinking

DOK 4 Extended thinking

For Math 2 teachers. Claims, targets and DOK levels follow the official Smarter Balanced blueprint.

achievementtestbank.com • October 2026 edition

How to use this booklet

Depth of Knowledge (DOK) describes the kind of thinking a question asks for, not how hard it is. A long calculation can still be DOK 1, and a short question can be DOK 3. For each target, this booklet shows one question at every DOK level SBAC assesses it at, so you can see the climb and then pick questions at the level your students need next.

The four DOK levels

DOK 1 • Recall

Recall a fact, term or property, or carry out a routine one-step procedure. Read a value straight from a table or graph.

DOK 2 • Skills & concepts

Decide how to approach it and use two or more steps: compare, classify, interpret data, or move between representations.

DOK 3 • Strategic thinking

Reason and plan: justify an answer, critique an argument, use evidence, or solve a problem with no routine path.

DOK 4 • Extended thinking

Carry out an extended investigation that connects several ideas, conditions or sources. On SBAC, DOK 4 is assessed only in the performance task, and only for Claim 3 and Claim 4 targets.

Reading a target page

1. The heading names the claim, the target code and the official target wording, with the standards it draws on.
2. The DOK ladder shows which levels the SBAC blueprint assesses for this target in this grade. Levels in grey with a line through them are not assessed for this target.
3. One example question at each assessed level, lowest first, with the answer shown. Read them top to bottom to see how the thinking climbs.
4. "Why DOK n" explains in one sentence what makes the question that level, so you can spot the same thing in your own questions.
5. "Find more like this" lists the exact Test Bank clicks that generate more questions for the same target at the same level.

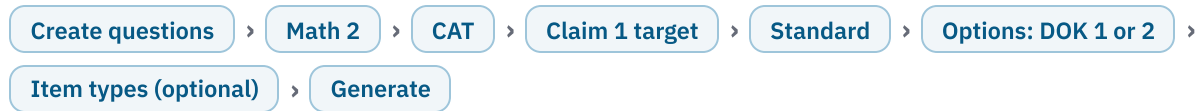
Which DOK levels each target is assessed at comes from the official Smarter Balanced test blueprint for this grade (2015–16 Summative Technical Report, Appendix 2.A). Every example was checked by hand against the DOK definitions above, not just its Test Bank tag. A few targets had no Test Bank question that truly fit a level, so we wrote one; those are marked "Written for this guide."

Finding these questions in the Test Bank

Every example page ends with the exact clicks. These are the two routes, starting from Create questions with Math 2 chosen as the course.

Claim 1 targets (CAT)

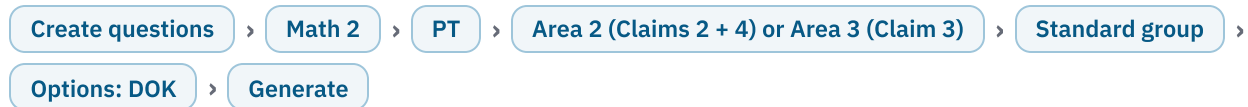
Claim 1 questions are filed by target and standard.



The DOK buttons only allow the levels the blueprint assesses for that target, so a level that is greyed out in this booklet is greyed out in the Test Bank too.

Claims 2, 3 and 4 (PT)

These claims are built from performance-task standard groups.



Each question card shows its claim and target, such as "Claim 3 · Tgt B". Generate at the DOK you want, then keep the cards for your target.

Questions you have saved

Saved questions are filed the same way in your Item Bank.



Tip: to move a class up the ladder, generate the same target at the next DOK level. To check that a skill is secure before going further, generate a set at the lowest assessed level.

All targets at a glance

A dot means the SBAC blueprint assesses the target at that DOK level in this grade. Each dot has an example question in this booklet.

Claim 1 • Concepts & Procedures

Target	DOK 1	DOK 2	DOK 3	DOK 4
1A Extend the properties of exponents to rational exponents	●	●	—	—
1B Use properties of rational and irrational numbers	●	●	—	—
1D Interpret the structure of expressions	●	●	—	—
1E Write expressions in equivalent forms to solve problems	●	●	—	—
1F Perform arithmetic operations on polynomials	—	●	—	—
1G Create equations that describe numbers or relationships	●	●	—	—
1I Solve equations and inequalities in one variable	●	●	—	—
1J Represent and solve equations and inequalities graphically	●	●	—	—
1L Interpret functions that arise in applications in terms of a context	●	●	—	—
1M Analyze functions using different representations	●	●	●	—
1N Build a function that models a relationship between two quantities	—	●	—	—
1O Define trigonometric ratios and solve problems involving right triangles	●	●	—	—
1P Summarize, represent, and interpret data on a single measurement variable	—	●	—	—

Claim 2 • Problem Solving

Target	DOK 1	DOK 2	DOK 3	DOK 4
2A Apply mathematics to solve well-posed problems from everyday life	—	●	●	—
2B Select and use appropriate tools strategically	●	●	●	—
2C Interpret results in the context of a situation	●	●	●	—
2D Identify important quantities in a practical situation and map their relationships (e.g., using diagrams, two-way tables, graphs, flow charts, or formulas)	●	●	●	—

Claim 3 • Communicating Reasoning

Target		DOK 1	DOK 2	DOK 3	DOK 4
3A	Test propositions or conjectures with specific examples	—	●	●	—
3B	Construct chains of reasoning that will justify or refute propositions	—	●	●	●
3C	State logical assumptions being used	—	●	●	—
3D	Use the technique of breaking an argument into cases	—	●	●	—
3E	Distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in the argument—explain what it is	—	●	●	●
3F	Base arguments on concrete referents such as objects, drawings, diagrams, and actions	—	●	●	—
3G	Determine conditions under which an argument does and does not apply	—	●	●	—

Claim 4 • Modeling & Data Analysis

Target		DOK 1	DOK 2	DOK 3	DOK 4
4A	Apply mathematics to solve problems from everyday life (Modeling)	—	●	●	—
4B	Construct chains of reasoning to justify mathematical models	—	●	●	●
4C	State logical assumptions being used	●	●	●	—
4D	Interpret results in the context of a situation	—	●	●	—
4E	Analyze adequacy and make improvements to existing models	—	●	●	●
4F	Identify important quantities in a practical situation and map their relationships (e.g., using diagrams, two-way tables, graphs, flow charts, or formulas)	●	●	●	—
4G	Identify, analyze, and synthesize relevant external resources to pose or solve problems	—	—	●	●

CLAIM 1

Claim 1 • Concepts & Procedures

Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency. Claim 1 is tested with individual items on the computer adaptive test (CAT). In the Test Bank, choose CAT, then the target.

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1A Extend the properties of exponents to rational exponents

Standards: N-RN.1; N-RN.2

Assessed at: **DOK 1** **DOK 2** DOK 3 DOK 4**DOK 1 · Recall****Why DOK 1:** Students apply the definition that a cube root equals the one-third power in a single step, which is recall of a rule rather than a decision about how to proceed.

A basketball player's vertical jump height can be modeled using the expression $\sqrt[3]{512}$. Which of the following expressions is equivalent to $\sqrt[3]{512}$ using rational exponents?

☒ $512^{1/3}$



☐ $512^{1/2}$

☐ 512^3

☐ 3^{512}

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Test Bank question

DOK 2 · Skills & concepts**Why DOK 2:** Each expression requires interpreting the exponent as both a root and a power and carrying out two steps in order, such as cube root then square, which moves beyond recall.

A basketball player's vertical jump height (in inches) can be modeled using rational exponents. Complete the table by rewriting each expression involving radicals or rational exponents in simplified form.

Original Expression	Simplified Form
$16^{1/2}$	4
$27^{2/3}$	9
$32^{3/5}$	8
$\sqrt[4]{81}$ or $81^{1/4}$	3

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Test Bank question

CLAIM 1 · CONCEPTS & PROCEDURES

1B Use properties of rational and irrational numbers

Standards: N-RN.3

Assessed at: **DOK 1** **DOK 2** DOK 3 DOK 4

DOK 1 · Recall

Why DOK 1: Students recall and state one property, that a rational plus an irrational number is irrational, and apply it directly to a single sum.

A basketball player's vertical jump height can be expressed as $3 + \sqrt{5}$ feet. Explain whether this height is a rational or irrational number. Use properties of rational and irrational numbers to justify your answer.

Model response: The height $3 + \sqrt{5}$ feet is irrational. Since 3 is a rational number and $\sqrt{5}$ is an irrational number, their sum must be irrational. According to the properties of rational and irrational numbers, the sum of a rational number and an irrational number is always irrational.

Rubric: 1 point: Correctly identifies the number as irrational and provides clear reasoning that the sum of a rational number (3) and an irrational number ($\sqrt{5}$) is irrational.

0 points: Anything less, including a response that is only partly correct.

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Test Bank question

DOK 2 · Skills & concepts

Why DOK 2: Students must first rationalize and simplify the expression, then classify the result using the product property, linking two steps together.

Use the drop-down menus to complete the statement about a runner's training data.

A coach records that a runner's average speed is $\frac{12}{\sqrt{3}}$ miles per hour. When rationalized, this

speed equals $4\sqrt{3}$ miles per hour. Since $4\sqrt{3}$ is irrational, the runner's

speed is a(n) irrational number.

Drop-down 1 choices: $\frac{4}{\sqrt{3}}$ $12\sqrt{3}$ **$4\sqrt{3}$ ✓**

Drop-down 2 choices: **irrational ✓** rational

Drop-down 3 choices: **irrational ✓** rational

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Test Bank question

1D Interpret the structure of expressions

Standards: A-SSE.1; A-SSE.2

Assessed at: **DOK 1** DOK 2 DOK 3 DOK 4

DOK 1 · Recall Why **DOK 1**: Students identify what each part of a given expression is (root, coefficient, constant term), which is recognizing terms and factors rather than reasoning.

A coach models a player's total season points with the expression $x^2 + 11x + 24$, which can be rewritten in factored form as $(x + 3)(x + 8)$. Decide whether each statement about this expression is True or False.

	True	False
The factor $(x + 3)$ shows that -3 is a root of the expression.	☒	☐
The factor $(x + 8)$ shows that -8 is a root of the expression.	☒	☐
In the expanded form $x^2 + 11x + 24$, the number 11 is a root of the expression.	☐	☒
The factored form $(x + 3)(x + 8)$ reveals the roots of the expression directly.	☒	☐
In the expanded form $x^2 + 11x + 24$, the number 24 is a factor multiplied by x .	☐	☒

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Test Bank question

Why DOK 2: Students compare three equivalent forms and interpret which structure shows the field's dimensions in context, connecting the factors to length and width.

A rectangular soccer field's area (in square yards) is modeled by $A(x) = x^2 + 10x + 21$, where x is a scale factor. A student rewrote the expression in three equivalent forms. Select the form that directly reveals the field's length and width as expressions in x .

Form 1: $x^2 + 10x + 21$

Form 2: $(x + 3)(x + 7)$ ✓

Form 3: $(x + 5)^2 - 4$

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Test Bank question

1E Write expressions in equivalent forms to solve problems

Standards: A-SSE.3

Assessed at: **DOK 1** **DOK 2** DOK 3 DOK 4**DOK 1 · Recall****Why DOK 1:** The vertex form is already given, so students read the maximum straight from the expression and recognize why it is the maximum.

A kicker punts a football during practice. The height of the ball in feet, x seconds after being kicked, is modeled by $h(x) = -x^2 + 8x + 9$. A coach rewrites the expression in the equivalent form $h(x) = -(x - 4)^2 + 25$ to analyze the punt.

Part A: What is the maximum height, in feet, that the football reaches?

Choose one answer.

☐ 9 feet☒ 25 feet ✓☐ 4 feet☐ 16 feetPart B: Which statement explains how the equivalent form $h(x) = -(x - 4)^2 + 25$ reveals the maximum height?

Choose one answer.

☐ The maximum occurs where $x = 25$, since that is the constant added in the completed square form☒ Since $-(x - 4)^2 \leq 0$ for all x , the maximum value of $h(x)$ occurs when $-(x - 4)^2 = 0$, giving $h(x) = 25$ ✓☐ Since $(x - 4)^2$ is always positive, $h(x)$ increases without bound as x increases☐ The constant term 9 in the original expression directly gives the maximum height

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Test Bank question

Why DOK 2: Students must carry out the multi-step completing-the-square process with fractions and then interpret the vertex in context.

A basketball player's height above the ground during a jump can be modeled by the quadratic function $h(t) = -16t^2 + 24t + 6$, where h is measured in feet and t is time in seconds. By completing the square, rewrite this function in vertex form $h(t) = a(t - h)^2 + k$. Then explain what the vertex tells you about the player's jump.

Model response: $h(t) = -16(t - \frac{3}{4})^2 + 15$. The vertex is at $(\frac{3}{4}, 15)$, which means the player reaches a maximum height of 15 feet at $\frac{3}{4}$ second (or 0.75 seconds) after jumping.

Rubric: 1 point: Correctly completes the square to obtain vertex form with accurate vertex coordinates, and clearly interprets both the time and maximum height in context.

0 points: Anything less, including a response that is only partly correct.

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Test Bank question

1F Perform arithmetic operations on polynomials

Standards: A-APR.1

Assessed at:

DOK 1

DOK 2

DOK 3

DOK 4

DOK 2 · Skills & concepts**Why DOK 2:** Students multiply binomials and must recognize several equivalent forms at different stages of the process, evaluating each choice against their work.

A rectangular soccer practice field has a length of $(x + 6)$ meters and a width of $(x + 4)$ meters. The area of the field is given by $(x + 6)(x + 4)$. Which of the following are equivalent expressions for the area of the field, in square meters? Select all that apply.

☒ $x^2 + 4x + 6x + 24$ ✓

☒ $x \cdot x + x \cdot 4 + 6 \cdot x + 6 \cdot 4$ ✓

☐ $x^2 + 24x + 10$

☐ $x^2 + 6x + 4x + 10$

☒ $x^2 + 10x + 24$ ✓

☐ $2x^2 + 10x + 24$

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Test Bank question

CLAIM 1 · CONCEPTS & PROCEDURES

1G Create equations that describe numbers or relationships

Standards: A-CED.1; A-CED.2; A-CED.4

Assessed at: **DOK 1** **DOK 2** DOK 3 DOK 4**DOK 1 · Recall****Why DOK 1:** Students translate a direct verbal description into an area equation, a routine setup with the relationships stated explicitly.

A rectangular soccer practice field has a length that is 5 yards more than its width. The area of the field is 84 square yards. Let w represent the width, in yards, of the field. Which equations correctly represent this relationship? Select all that apply.

☒ $w^2 + 5w = 84$ ✓

☒ $w(w + 5) = 84$ ✓

☐ $2w + 5 = 84$

☐ $w(w + 5) = 42$

☐ $w^2 - 5w = 84$

☒ $w^2 + 5w - 84 = 0$ ✓

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Test Bank question

DOK 2 · Skills & concepts**Why DOK 2:** Students must build a linear demand equation from two data points, then multiply by price to create the revenue model, planning two linked steps.

A volleyball team assumes ticket demand is linear in price. It sells 150 tickets at 12 dollars each and 120 tickets at 15 dollars each. Revenue equals price times demand. Write the revenue model as $R(p) = ap^2 + bp + c$. Enter only the coefficient b as a number.

Correct answer: **270**

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Test Bank question

1I Solve equations and inequalities in one variable

Standards: A-REI.4

Assessed at:

DOK 1

DOK 2

DOK 3

DOK 4

DOK 1 · Recall

Why DOK 1: Students solve a quadratic by factoring out a common factor, a routine one-procedure task, and check simple statements about the solutions.

A basketball player's jump height can be modeled by the equation $h(t) = -16t^2 + 32t$, where h is height in feet and t is time in seconds. Determine whether each statement about solving $-16t^2 + 32t = 0$ is true or false.

	True	False
The equation factors as $-16t(t - 2) = 0$.	☒	☐
One solution is $t = 2$ seconds.	☒	☐
The solutions are $t = 0$ and $t = 32$.	☐	☒
The solution $t = 0$ represents the moment the player leaves the ground.	☒	☐

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Test Bank question

DOK 2 · Skills & concepts

Why DOK 2: Students must work through the quadratic formula themselves and compare each step to find where the discriminant was miscalculated, which requires analyzing a procedure.

A punter kicks a football. The horizontal distance traveled satisfies the equation $x^2 - 6x - 7 = 0$, where x is measured in yards. Kai used the quadratic formula to solve for x . Select the step where Kai made his first error.

Step 1: $x^2 - 6x - 7 = 0$, so $a = 1$, $b = -6$, $c = -7$

Step 2: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{6 \pm \sqrt{(-6)^2 - 4(1)(-7)}}{2(1)}$

Step 3: $x = \frac{6 \pm \sqrt{36 - 28}}{2} = \frac{6 \pm \sqrt{8}}{2}$ ✓

Step 4: $x = \frac{6 \pm \sqrt{64}}{2} = \frac{6 \pm 8}{2}$

Step 5: $x = 7$ or $x = -1$

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Test Bank question

1J Represent and solve equations and inequalities graphically

Standards: A-REI.7

Assessed at: **DOK 1** **DOK 2** DOK 3 DOK 4**DOK 1 · Recall**

Why DOK 1: Students substitute a point into two equations to verify it satisfies both, a routine check that requires no choice of strategy.

Which ordered pair is a solution to the system of equations $y = x^2 - 3$ and $y = 2x$?

☒ (3, 6) ✓☐ (1, -2)☐ (2, 4)☐ (-3, 6)

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A basketball player's height above the ground during a jump can be modeled by $h(t) = -16t^2 + 20t + 6$, where h is height in feet and t is time in seconds. The rim is at a constant height of $h(t) = 10$ feet. By looking at a graph of both functions, explain how you would find the time(s) when the player's hand reaches the rim height. What does the x-coordinate of each intersection point represent?

Model response: To find when the player's hand reaches the rim, I would graph both $h(t) = -16t^2 + 20t + 6$ (the parabola representing the player's hand) and $h(t) = 10$ (the horizontal line representing the rim). The x-coordinates of the intersection points are the solutions to $-16t^2 + 20t + 6 = 10$, which simplifies to $-16t^2 + 20t - 4 = 0$. Each x-coordinate represents the time (in seconds) when the player's hand is at rim height—once on the way up and once on the way down.

Rubric: 1 point: Correctly identifies that intersection points represent solutions to the equation, accurately explains that x-coordinates are times when heights are equal, and describes the physical meaning (rising and falling).

0 points: Anything less, including a response that is only partly correct.

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Test Bank question

1L Interpret functions that arise in applications in terms of a context

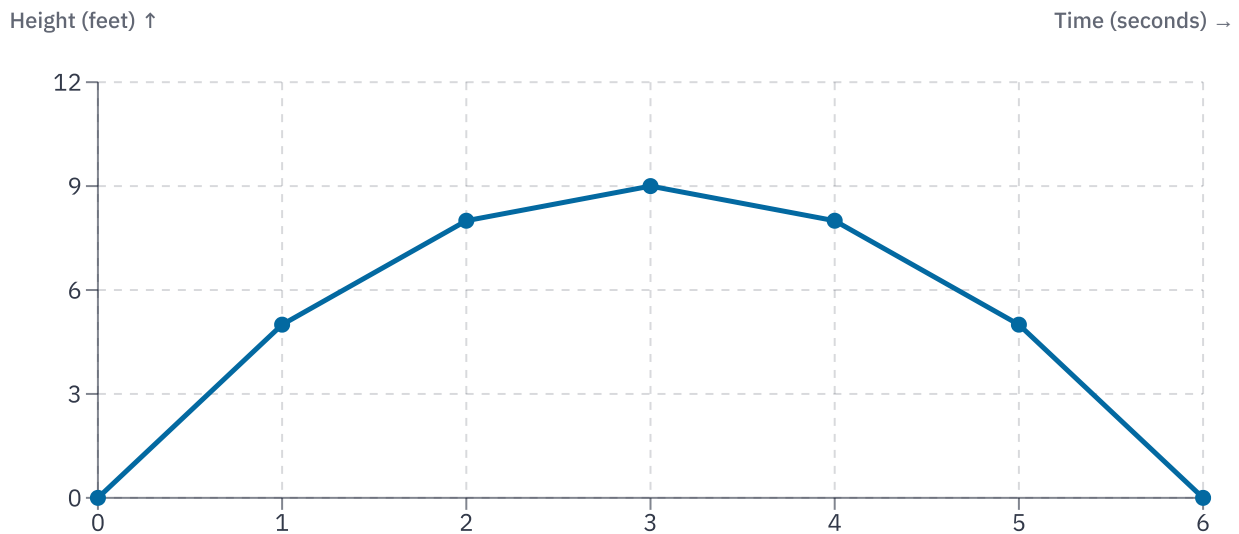
Standards: F-IF.4; F-IF.5; F-IF.6

Assessed at: **DOK 1** **DOK 2** DOK 3 DOK 4**DOK 1 · Recall**

Why DOK 1: Students read the highest point directly off the graph, which is locating a value rather than interpreting or calculating.

The graph shows the height of a basketball, in feet, t seconds after it is thrown. Based on the graph, what is the maximum height of the basketball, and at what time does it occur?

Height of a Basketball Free Throw Over Time



- ☐ The maximum height is 9 feet, occurring at $t = 6$ seconds
- ☐ The maximum height is 8 feet, occurring at $t = 2$ seconds
- ☐ The maximum height is 6 feet, occurring at $t = 6$ seconds
- ☒ The maximum height is 9 feet, occurring at $t = 3$ seconds ✓

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Test Bank question

A soccer ball is kicked into the air. Its height above the ground, in feet, is modeled by $h(t) = -16t^2 + 64t + 4$, where t is the time in seconds after the kick. Based on this function, which statements are true? Select all that apply.

☐ The average rate of change in height from $t = 0$ to $t = 2$ seconds is 32 feet per second. ✓

☐ The average rate of change in height from $t = 2$ to $t = 4$ seconds is 32 feet per second.

☐ The ball was kicked from an initial height of 4 feet above the ground. ✓

☐ The ball reaches a maximum height of 68 feet, occurring at $t = 2$ seconds. ✓

☐ The ball was kicked from ground level, so $h(0) = 0$.

☐ The ball reaches its maximum height at $t = 4$ seconds.

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Test Bank question

CLAIM 1 • CONCEPTS & PROCEDURES

1M Analyze functions using different representations

Standards: F-IF.7; F-IF.8; F-IF.9; F-BF.3; F-BF.4a

Assessed at: DOK 1 DOK 2 DOK 3 DOK 4

DOK 1 • Recall

Why DOK 1: With the vertex form given, students simply read the maximum height and its time from the expression, a direct recall of what vertex form shows.

Use the drop-down menus to complete the statement.

A basketball player's height above the ground during a jump can be modeled by $h(t) = -16t^2 + 32t$, where h is in feet and t is time in seconds. The vertex form of this function is $h(t) = -16(t - 1)^2 + 16$. Based on these two representations, the maximum height reached is 16 feet, and this maximum occurs at $t = 1$ seconds.

Drop-down 1 choices: 8 16 ✓ 32

Drop-down 2 choices: $t = 2$ $t = 1$ ✓ $t = 0.5$

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Test Bank question

DOK 2 • Skills & concepts

Why DOK 2: Students extract the vertex and zeros from two representations and explain what each representation makes easiest to see, comparing features.

A basketball player's height above the ground during a jump is modeled by two different representations:

Equation (vertex form): $h(t) = -16(t - 0.5)^2 + 4$

Table (selected values):

Time (seconds)	Height (feet)
0	0
0.5	4
1	0

where t is time in seconds and $h(t)$ is height in feet.

Identify the vertex, the time(s) when the player is at ground level, and explain what each representation makes easiest to see. Why would you choose vertex form over the table to analyze this jump?

Model response: Vertex: The vertex is $(0.5, 4)$, meaning the player reaches a maximum height of 4 feet at 0.5 seconds.

Times at ground level: The player is at ground level at $t = 0$ and $t = 1$ second. These are visible directly in the table.

What each representation shows best:

- The **vertex form equation** $h(t) = -16(t - 0.5)^2 + 4$ immediately reveals the maximum height (4 feet) and when it occurs (0.5 seconds), as well as the symmetry of the jump.
- The **table** clearly shows specific input-output pairs, making the ground-level times easy to read without calculation.

Why vertex form is more useful: Vertex form directly shows the extreme value (maximum height) and its timing without needing to convert or calculate further. This is strategic when analyzing the peak of a jump, whereas the table only shows discrete points and requires inference or calculation to identify the true maximum.

Rubric: 1 point: Student correctly identifies vertex as $(0.5, 4)$ and ground-level times as $t = 0$ and $t = 1$; clearly explains what each representation reveals; provides a thoughtful comparison of why vertex form is more efficient for analyzing the jump's maximum.

0 points: Anything less, including a response that is only partly correct.

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Test Bank question

DOK 3 · Strategic thinking

Why DOK 3: Students must plan a path through two representations, build an equation from the table, compare features, and justify conclusions, which goes well beyond reading a vertex.

Two balls are launched during a physics lab. Ball A's height, in feet, t seconds after launch is $h_A(t) = -16t^2 + 48t + 4$. Ball B's height was recorded in a table:

t (seconds)	0	0.5	1	1.5	2	2.5
$h_B(t)$ (feet)	6	26	38	42	38	26

Ball B's height also follows a quadratic function.

- Which ball reaches the greater maximum height?
- Which ball hits the ground first?

Justify each answer using features of the two representations, and show your work.

Model response: (a) For Ball A, the axis of symmetry is $t = -\frac{48}{2(-16)} = 1.5$, so its maximum height is $h_A(1.5) = -36 + 72 + 4 = 40$ feet. Ball B's table is symmetric about $t = 1.5$ (38 and 38, 26 and 26), so its vertex is $(1.5, 42)$ and its maximum height is 42 feet. Ball B reaches the greater maximum height.

(b) Using the vertex $(1.5, 42)$ and the point $(0, 6)$: $6 = a(0 - 1.5)^2 + 42$, so $a = -16$ and $h_B(t) = -16(t - 1.5)^2 + 42$. Ball B lands when $(t - 1.5)^2 = \frac{42}{16} = 2.625$, so $t = 1.5 + \sqrt{2.625} \approx 3.12$ seconds. Ball A in vertex form is $h_A(t) = -16(t - 1.5)^2 + 40$, so it lands when $(t - 1.5)^2 = 2.5$, giving $t = 1.5 + \sqrt{2.5} \approx 3.08$ seconds. Ball A hits the ground first: both balls peak at the same time and fall the same way, but Ball A starts its fall from a lower maximum.

Rubric: 1 point: Finds both maximum heights (40 ft and 42 ft) and both landing times (about 3.08 s and 3.12 s), concludes that Ball B goes higher and Ball A lands first, and justifies each conclusion with features of the representations (vertex, symmetry of the table, equation built from the table).

0 points: Anything less, including a response that is only partly correct.

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CLAIM 1 · CONCEPTS & PROCEDURES**1N Build a function that models a relationship between two quantities****Standards:** F-BF.1; F-LE.3; F-LE.6

Assessed at:

DOK 1

DOK 2

DOK 3

DOK 4

DOK 2 · Skills & concepts**Why DOK 2:** Students use the vertex and a second point to find the leading coefficient and write the function, building a model from given features in several steps.

A lacrosse player kicks a ball from ground level. The height of the ball above the ground (in feet) can be modeled as a function of time (in seconds) after the kick. The ball reaches a maximum height of 64 feet at 2 seconds, and returns to ground level at 4 seconds.

Write a quadratic function $h(t)$ that models the height of the ball. Use the vertex form $h(t) = a(t - h)^2 + k$, where the vertex is (h, k) . Expand your answer to standard form $h(t) = at^2 + bt + c$.

Write your answer in the form $h(t) = at^2 + bt + c$.

Correct answer: $h(t) = -16t^2 + 64t$

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Test Bank question

10 Define trigonometric ratios and solve problems involving right triangles

Standards: G-SRT.6; G-SRT.7; G-SRT.8; G-SRT.8.1

Assessed at: **DOK 1** **DOK 2** DOK 3 DOK 4

DOK 1 · Recall

Why DOK 1: Students identify which trigonometric ratio links the opposite and adjacent sides, recalling a definition in a single step.

A basketball player is standing 12 feet away from the base of the hoop. The angle of elevation from the player's eyes to the center of the hoop is 35° . Which trigonometric ratio should be used to find the height of the hoop above the player's eyes?

☒ $\tan(35^\circ) = \frac{\text{height}}{12}$ ✓

☐ $\tan(35^\circ) = \frac{12}{\text{height}}$

☐ $\sin(35^\circ) = \frac{\text{height}}{12}$

☐ $\cos(35^\circ) = \frac{\text{height}}{12}$

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Test Bank question

Use the drop-down menus to complete the statement.

A volleyball player is practicing spike serves. The ball is hit from a height of 2 meters above the court, and it travels horizontally a distance of 8 meters before hitting the ground. The angle of depression from the player's hand to the point where the ball lands is

approximately degrees. (Round to the nearest whole degree.) To find this

angle, the player uses the ratio $\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$.

Drop-down 1 choices: ✓

Drop-down 2 choices: $\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$ ✓ $\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}}$ $\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}}$

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Test Bank question

CLAIM 1 • CONCEPTS & PROCEDURES

1P Summarize, represent, and interpret data on a single measurement variable

Standards: S-ID.6

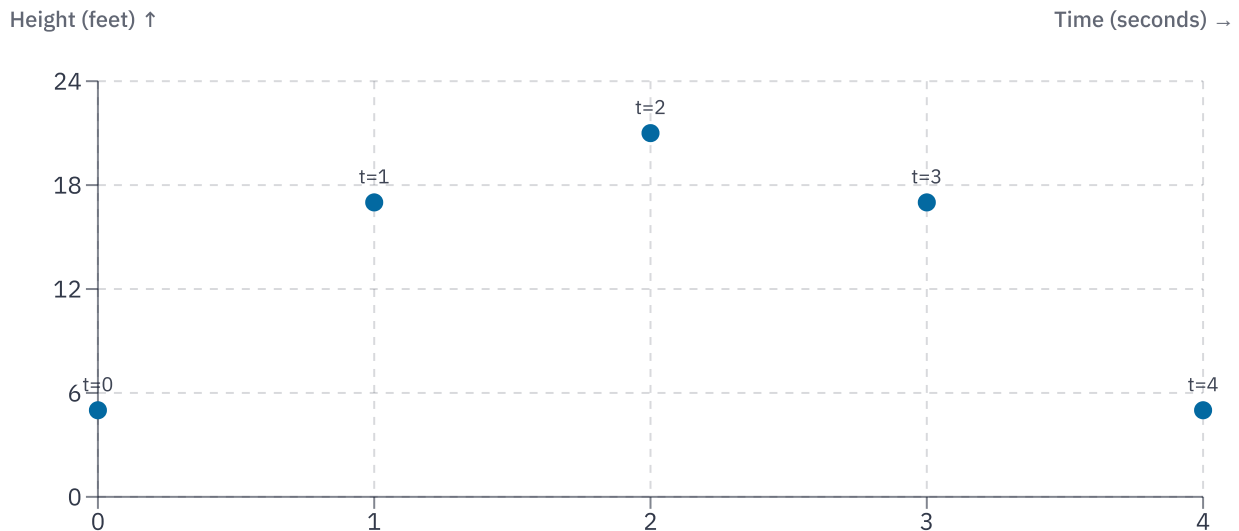
Assessed at: DOK 1 **DOK 2** DOK 3 DOK 4

DOK 2 • Skills & concepts

Why DOK 2: Students use features of the plotted data (initial value, vertex, symmetry) to select and verify the quadratic function that fits, linking data to an equation.

A basketball is thrown into the air. The scatter plot shows the ball's height (in feet) at several times (in seconds) during its flight. The path of the ball can be modeled by a quadratic equation of the form $h(t) = at^2 + bt + c$, where $h(t)$ is the height in feet and t is the time in seconds. Based on the pattern of the data (initial height of 5 feet, maximum height of 21 feet at $t = 2$, and symmetric heights of 17 feet at $t = 1$ and $t = 3$), which equation correctly models the height of the ball?

Height of a Basketball Shot Over Time



☐ $h(t) = 4t^2 + 16t + 5$

☐ $h(t) = -2t^2 + 16t + 5$

☐ $h(t) = -4t^2 + 8t + 5$

☒ $h(t) = -4t^2 + 16t + 5$ ✓

FIND MORE LIKE THIS

CLAIM 2

Claim 2 • Problem Solving

Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem-solving strategies. In the Test Bank, Claims 2, 3 and 4 are built in the performance task (PT) step: choose PT, then Area 2 • Claims 2 + 4. Each question card is marked with its claim and target (for example "Claim 2 • Tgt A"), so generate a set at the DOK you want and keep the cards for the target you need.

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> Area 2 • Claims 2 + 4

> Standard group

> Options: DOK

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2A Apply mathematics to solve well-posed problems from everyday life

Assessed at: DOK 1 **DOK 2** DOK 3 DOK 4**DOK 2 • Skills & concepts**

Why DOK 2: Students solve a well-posed problem by isolating the rational-exponent term and raising both sides to the reciprocal power, a two-step process with a clear path.

On a family road trip, the number of gallons of gas used, G , is modeled by $G = d^{2/3}/10$, where d is the distance traveled in miles. The family used 6.4 gallons of gas on one leg of the trip. Solve for d , the distance traveled in miles.

Correct answer: **512**

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Test Bank question

DOK 3 · Strategic thinking

Why DOK 3: Students must model the situation with right triangles, choose the right ratios, compare results to constraints, and evaluate a redesign, which takes planning across several steps.

A school is adding a wheelchair ramp to a stage that is 3 feet high. Building rules say the ramp may make an angle of at most 4.8° with the floor. The space in front of the stage allows at most 32 feet of horizontal run.

- (a) Can a single straight ramp fit in the space? Show the calculation that supports your answer.
- (b) The designer instead plans a ramp with two straight sections that each rise half the height of the stage, connected by a flat landing, with the second section turning back alongside the first. Each section must fit within 20 feet of horizontal run. Using the steepest allowed angle, does this design work? Find the length of the ramp surface for each section, to the nearest tenth of a foot.

Model response: (a) No. The ramp is the hypotenuse of a right triangle with a vertical rise of 3 ft. At the steepest allowed angle, the horizontal run is $\frac{3}{\tan 4.8^\circ} \approx \frac{3}{0.0840} \approx 35.7$ feet. That is more than the 32 feet available, and any smaller angle would need an even longer run, so a single straight ramp does not fit.

(b) Yes. Each section rises 1.5 ft. At 4.8° , each section needs a horizontal run of $\frac{1.5}{\tan 4.8^\circ} \approx 17.9$ feet, which is less than 20 feet, so each section fits. The ramp surface for each section is $\frac{1.5}{\sin 4.8^\circ} \approx 17.9$ feet (about 17.93 ft).

Rubric: 1 point: Correctly shows the single ramp needs about 35.7 ft of run and does not fit, shows each section of the two-part design needs about 17.9 ft of run and fits, and finds each section's surface length of about 17.9 ft, with appropriate trigonometric ratios.

0 points: Anything less, including a response that is only partly correct.

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2B Select and use appropriate tools strategically

 Assessed at: **DOK 1** **DOK 2** **DOK 3** **DOK 4**
DOK 1 • Recall

Why DOK 1: Students recognize whether a single known rule applies to each task, a quick yes-or-no identification without carrying out a strategy.

A coach is using a calculator and algebraic rules to convert between radical and exponential forms when analyzing player performance metrics. For each expression, decide whether using the property $a^{m/n} = \sqrt[n]{a^m}$ (or its inverse) is the most appropriate tool to evaluate or simplify it. Select Yes or No for each.

	Yes, use rational exponent property	No, use a different approach
Simplifying $16^{1/2}$ to find a player's speed rating	☒	☐
Rewriting $\sqrt[3]{27}$ in exponential form as $27^{1/3}$	☒	☐
Finding the value of $8^{2/3}$ to calculate a fitness score	☒	☐
Adding the two expressions $3x + 5$ and $2x + 7$	☐	☒
Converting $\sqrt[4]{x^3}$ to $x^{3/4}$ for a performance model	☒	☐

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Test Bank question

DOK 2 · Skills & concepts

Why DOK 2: Students compare four solution approaches and judge which tool gives an exact value most efficiently, which requires weighing methods rather than recalling one.

A sports analyst needs the exact value of a player's power rating, given by $P = 32^{3/5}$. Four students describe how they solved this. Select the step that shows the most strategic and efficient use of mathematical tools to find the exact value.

Step 1: Typed $32^{3/5}$ into a calculator and rounded the decimal result to two places, reporting $P \approx 8.00$.

Step 2: Rewrote $32^{3/5}$ as $(32^{1/5})^3$, recognized $32^{1/5} = 2$ since $2^5 = 32$, then computed $2^3 = 8$ to get the exact value $P = 8$. ✓

Step 3: Guessed and checked whole numbers, testing $5^{3/5}$, $6^{3/5}$, and $7^{3/5}$ on a calculator until one was close to a whole number.

Step 4: Graphed $y = 32^x$ on a graphing calculator and traced the curve near $x = 3/5$ to estimate the y -value.

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Test Bank question

DOK 3 · Strategic thinking

Why DOK 3: Students must evaluate which tools fit the problem, use the discriminant to justify rejecting factoring, solve with an appropriate tool, and verify with another, which requires strategic decisions.

A ball is tossed upward. Its height, in feet, t seconds after it is tossed is $h(t) = -16t^2 + 40t + 5$. Three students plan how to find when the ball hits the ground.

- Ana: factor $-16t^2 + 40t + 5$ and use the zero product property.
- Ben: use the quadratic formula.
- Cam: graph $h(t)$ on a graphing calculator and find the positive t -intercept.

Decide which method or methods will work, and explain why any method will not work well here. Then find the time the ball hits the ground, to the nearest hundredth of a second, and explain how one of the other methods could be used to check your answer.

Model response: Setting $h(t) = 0$ gives $-16t^2 + 40t + 5 = 0$, or $16t^2 - 40t - 5 = 0$. The discriminant is $(-40)^2 - 4(16)(-5) = 1600 + 320 = 1920$, which is not a perfect square, so the expression does not factor over the integers. Ana's factoring method will not work well. Ben's quadratic formula works and gives exact answers: $t = \frac{40 \pm \sqrt{1920}}{32} = \frac{5 \pm \sqrt{30}}{4}$. The negative root does not make sense for time, so the ball lands at $t = \frac{5 + \sqrt{30}}{4} \approx 2.62$ seconds. Cam's graphing method also works but gives an approximate value; the graph's positive t -intercept near 2.62 can be used to check the answer from the formula.

Rubric: 1 point: Explains why factoring does not work (for example, by showing the discriminant 1920 is not a perfect square), identifies the quadratic formula and graphing as workable, finds $t \approx 2.62$ seconds while rejecting the negative root, and describes how graphing (or substitution) checks the result.

0 points: Anything less, including a response that is only partly correct.

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2C Interpret results in the context of a situation

Assessed at: **DOK 1** **DOK 2** **DOK 3** ~~DOK 4~~**DOK 1 • Recall****Why DOK 1:** Students identify what each factor and coefficient represents in context and check simple statements, a direct interpretation of given parts.

A sports arena's revenue from selling tickets to a basketball game is modeled by $R(x) = -5x^2 + 300x$, where x is the ticket price in dollars and $R(x)$ is revenue in dollars. This expression can be rewritten in factored form as $R(x) = x(300 - 5x)$. Decide whether each statement about this model is True or False.

	True	False
In the factored form $R(x) = x(300 - 5x)$, the factor x represents the ticket price in dollars.	☒	☐
In the factored form $R(x) = x(300 - 5x)$, the factor $(300 - 5x)$ represents the number of tickets sold at price x .	☒	☐
The negative coefficient -5 in $-5x^2$ means that revenue always decreases as the ticket price increases, for every value of x .	☐	☒
Setting $R(x) = 0$ shows that revenue is zero when the ticket price is $x = 0$ dollars or $x = 60$ dollars.	☒	☐
Maximum revenue occurs when $300 - 5x = 0$, that is, when the ticket price is $x = 300$ dollars.	☐	☒

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Test Bank question

A basketball team's scoring model estimates points per game as $P(x) = -2x^2 + 40x + 15$, where x represents the number of hours spent in practice per week. The team currently practices 8 hours per week. The coach is considering increasing practice to 10 hours per week. Calculate the predicted points per game at both practice levels, then explain what this change tells you about the relationship between practice time and scoring, including any concerns about continuing to increase practice indefinitely.

Model response: At $x = 8$: $P(8) = -2(8)^2 + 40(8) + 15 = -128 + 320 + 15 = 207$ points per game. At $x = 10$: $P(10) = -2(10)^2 + 40(10) + 15 = -200 + 400 + 15 = 215$ points per game. The increase of 8 points shows that more practice improves scoring in this range. However, the negative leading coefficient (-2) indicates this is a downward-opening parabola with a maximum around $x = 10$ hours. Beyond this point, additional practice would actually decrease scoring, suggesting diminishing returns or potential fatigue and overtraining. This model suggests there is an optimal practice duration rather than a linear benefit to more practice.

Rubric: 1 point: Correctly calculates both values, identifies the 8-point increase, and explains that the negative coefficient creates a maximum point beyond which scoring declines—demonstrating understanding of polynomial structure and practical implications.

0 points: Anything less, including a response that is only partly correct.

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Test Bank question

DOK 3 · Strategic thinking

Why DOK 3: Students compute two predictions and then must judge each against the data range used to build the model, justifying which result is meaningful.

A track coach studies how standing vertical jump height relates to body mass using the rational-exponent model $H(m) = 120 m^{-1/3}$, where $H(m)$ is predicted jump height in centimeters and m is body mass in kilograms. The coach built this model using data from adult sprinters with masses between 50 kg and 100 kg. Use the properties of rational exponents to find the exact values of $H(64)$ and $H(8)$. Then explain whether each result is a meaningful prediction in the context of the coach's athletes, discussing the model's range of validity and your confidence in each prediction. Explain your reasoning.

Model response: Since $64^{1/3} = 4$, $H(64) = 120/4 = 30$ cm. Since $8^{1/3} = 2$, $H(8) = 120/2 = 60$ cm. The value $H(64) = 30$ cm is a reasonable, trustworthy prediction because 64 kg falls within the 50–100 kg range of the sprinters whose data built the model. However, $H(8) = 60$ cm is not a meaningful prediction: no adult sprinter has a mass of 8 kg, so this input lies far outside the data used to create the model. Using the formula there is an extrapolation beyond the model's valid domain, so the result, while mathematically correct, has no real-world confidence and should not be interpreted as an actual jump height.

Rubric: 1 point: Correctly computes $H(64) = 30$ cm and $H(8) = 60$ cm using rational exponent properties, and clearly explains that the first is a valid prediction within the model's data range while the second is an invalid extrapolation outside the realistic mass domain, discussing prediction confidence and model limitations.

0 points: Anything less, including a response that is only partly correct.

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Test Bank question

CLAIM 2 • PROBLEM SOLVING

2D Identify important quantities in a practical situation and map their relationships (e.g., using diagrams, two-way tables, graphs, flow charts, or formulas)Assessed at: **DOK 1** **DOK 2** **DOK 3** ~~DOK 4~~**DOK 1 • Recall****Why DOK 1:** Students match an equation to the table that represents it by evaluating simple cube roots, identifying the relationship between two quantities directly.

A soccer coach is analyzing player performance. She measures how a player's running speed relates to the oxygen uptake required. The relationship is modeled by the equation $s = o^{1/3}$, where s is the player's speed in meters per second and o is the oxygen uptake in milliliters per kilogram per minute. Which table correctly shows the relationship between the quantities in this model?

☐ Oxygen uptake (o): 2, 3, 4 | Speed (s): 8, 27, 64☐ Oxygen uptake (o): 8, 27, 64 | Speed (s): 4, 9, 16☒ Oxygen uptake (o): 8, 27, 64 | Speed (s): 2, 3, 4 ✓☐ Oxygen uptake (o): 1, 2, 3 | Speed (s): 2, 3, 4

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Test Bank question

DOK 2 · Skills & concepts

Why DOK 2: Students identify independent and dependent quantities, rewrite the rational exponent as a radical, and compute the output, connecting several pieces of the relationship.

A sports scientist uses the formula $P = 4M^{3/2}$ to estimate an athlete's maximum sprint power output P (in watts) based on the athlete's body mass M (in kilograms). Identify which quantity is the independent variable and which is the dependent variable, rewrite the expression $M^{3/2}$ using radical notation to show what it means, and then find the power output for an athlete with a mass of 64 kg. Explain your reasoning.

Model response: Mass M is the independent variable (it is chosen/measured first), and power P is the dependent variable because it is calculated from M . The rational exponent means $M^{3/2} = (\sqrt{M})^3 = \sqrt{M^3}$, so raising M to the $3/2$ power is the same as taking the square root of M then cubing the result. For $M = 64$: $\sqrt{64} = 8$, and $8^3 = 512$, so $M^{3/2} = 512$. Then $P = 4(512) = 2048$ watts.

Rubric: 1 point: Correctly identifies M as independent and P as dependent, correctly rewrites $M^{3/2}$ as $(\sqrt{M})^3$ or $\sqrt{M^3}$, and computes $P = 2048$ watts with clear reasoning connecting the quantities.
0 points: Anything less, including a response that is only partly correct.

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Test Bank question

DOK 3 · Strategic thinking

Why DOK 3: Students must identify several linked quantities (outer dimensions, walkway area, cost), map them into one equation, solve it, and judge which solution fits, which requires planning.

A rectangular garden is 10 feet long and 6 feet wide. A walkway of uniform width x feet will be built around the garden. The walkway material costs 3 dollars per square foot, and the budget for material is 240 dollars.

- (a) Write expressions in terms of x for the outer length, the outer width, and the area of the walkway only.
- (b) Write and solve an equation or inequality to find the greatest walkway width the budget allows.
- (c) Explain why any other solution of your equation does not make sense in this situation.

Model response: (a) Outer length: $10 + 2x$ feet. Outer width: $6 + 2x$ feet. Walkway area: $(10 + 2x)(6 + 2x) - 60 = 4x^2 + 32x$ square feet.

(b) Cost: $3(4x^2 + 32x) \leq 240$, so $4x^2 + 32x \leq 80$, or $x^2 + 8x - 20 \leq 0$. Solving $x^2 + 8x - 20 = 0$ gives $(x + 10)(x - 2) = 0$, so $x = 2$ or $x = -10$. The greatest walkway width the budget allows is 2 feet. Check: the walkway area is $14 \cdot 10 - 60 = 80$ square feet, which costs $3 \cdot 80 = 240$ dollars.

(c) $x = -10$ would mean a negative walkway width, which is impossible, so it is rejected.

Rubric: 1 point: Writes correct expressions for the outer length, outer width, and walkway area; sets up and solves a correct equation or inequality to get a width of 2 feet; and explains why $x = -10$ is rejected.

0 points: Anything less, including a response that is only partly correct.

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CLAIM 3

Claim 3 • Communicating Reasoning

Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others. In the Test Bank, Claims 2, 3 and 4 are built in the performance task (PT) step: choose PT, then Area 3 • Claim 3. Each question card is marked with its claim and target (for example "Claim 3 • Tgt A"), so generate a set at the DOK you want and keep the cards for the target you need.

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3A Test propositions or conjectures with specific examples

 Assessed at: DOK 1 **DOK 2** DOK 3 DOK 4
DOK 2 • Skills & concepts

Why DOK 2: Students test each given statement with numbers to decide whether a counterexample exists, applying exponent properties to several specific cases.

A coach models the relationship between a sprinter's training intensity and performance using rational exponents. For each statement about rational exponent properties, test it with specific numerical examples. Determine whether the statement is always true or whether you can find a counterexample that makes it false.

	Always True	Can Be False (Counterexample Exists)
For any positive number x , the expression $x^{1/2} \cdot x^{1/3}$ equals $x^{5/6}$.	☒	☐
For any positive number x , we have $(x^{1/2})^2 = x$.	☒	☐
For any positive numbers x and y , the expression $(xy)^{1/3}$ equals $x^{1/3} + y^{1/3}$.	☐	☒
For any positive number x , the radical expression $\sqrt[4]{x^3}$ can be rewritten as $x^{3/4}$ using rational exponents.	☒	☐
For any positive number x , if $x^{1/2} = 4$, then $x = 2$.	☐	☒

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Test Bank question

DOK 3 · Strategic thinking

Why DOK 3: Students must choose strategic test values from different cases, evaluate them, and generalize a pattern to decide the conjecture, which requires reasoning and planning.

A student claims that for any positive number a , the expression $a^{1/2}$ is always smaller than $a^{1/3}$. Test this conjecture with at least three specific examples, choosing values that represent different cases (such as values between 0 and 1, and values greater than 1). Based on your tests, determine whether the claim is true or false, and explain what your examples show.

Model response: Testing $a = 8$: $8^{1/2} = \sqrt{8} \approx 2.83$ and $8^{1/3} = \sqrt[3]{8} = 2$. Since $2.83 > 2$, the claim is false for $a = 8$. Testing $a = 0.5$: $(0.5)^{1/2} \approx 0.71$ and $(0.5)^{1/3} \approx 0.79$. Here $0.71 < 0.79$, so the claim is true for $a = 0.5$. Testing $a = 1$: $1^{1/2} = 1$ and $1^{1/3} = 1$, so they are equal. The claim is false because it is not always true—when $a > 1$, we have $a^{1/2} > a^{1/3}$, which contradicts the conjecture. The examples show that whether $a^{1/2}$ or $a^{1/3}$ is larger depends on whether a is greater than 1 or between 0 and 1.

Rubric: 1 point: Student tests at least three strategic examples (including values greater than 1, between 0 and 1, and/or the boundary case $a = 1$), correctly evaluates each using properties of rational exponents, identifies the claim as false, and explains the pattern discovered (that the relationship reverses depending on the value of a).

0 points: Anything less, including a response that is only partly correct.

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Test Bank question

3B Construct chains of reasoning that will justify or refute propositions

Assessed at: DOK 1 DOK 2 DOK 3 DOK 4 DOK 4 is assessed only in the performance task.

DOK 2 • Skills & concepts

Why DOK 2: Students expand the proposed form and compare it term by term to choose the justification that actually proves the claim wrong.

A bakery's weekly profit from selling bread can be modeled by the polynomial expression $P(x) = 45x^2 + 180x + 135$, where x represents the number of extra loaves baked. A student claims that this expression can be rewritten as $P(x) = 45(x + 2)^2 + 45$. Which of the following gives the most rigorous justification for whether this claim is correct or incorrect?

- ☐ The claim is correct because factoring 45 from the original expression gives $45(x^2 + 4x + 3)$, which is another valid way to write $45(x + 2)^2 + 45$.
- ☐ The claim is incorrect because the original expression starts with $45x^2$ while the rewritten form starts with $45(x + 2)^2$, and these two leading terms cannot represent the same variable.
- ☒ The claim is incorrect because expanding $45(x + 2)^2 + 45$ gives $45x^2 + 180x + 225$, which has a constant term of 225 instead of 135, so the two expressions are not equal. ✓
- ☐ The claim is correct because both expressions share the terms $45x^2$ and $180x$, so their constant terms must also automatically match.

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Test Bank question

DOK 3 · Strategic thinking

Why DOK 3: Students must build a general argument linking the sign of each part of the discriminant to the number of solutions, rather than testing examples, which is strategic reasoning.

Jada makes this claim: "If c is a negative number, then the equation $x^2 + bx + c = 0$ always has two different real solutions, no matter what real number b is."

Construct a chain of reasoning to justify or refute Jada's claim. Your argument must work for every real value of b , not just a few examples.

Model response: The claim is true. Whether $x^2 + bx + c = 0$ has real solutions depends on the discriminant $b^2 - 4ac$, which here is $b^2 - 4(1)(c) = b^2 - 4c$. For any real number b , $b^2 \geq 0$. Because $c < 0$, the term $-4c$ is positive. A nonnegative number plus a positive number is positive, so $b^2 - 4c > 0$ for every real b . A positive discriminant means the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4c}}{2}$ gives two different real values. Therefore, whenever c is negative, the equation has two different real solutions, no matter what b is. (Check: $b = 0$, $c = -4$ gives $x = \pm 2$.)

Rubric: 1 point: States that the claim is true and gives a complete general argument: the discriminant is $b^2 - 4c$, $b^2 \geq 0$, $-4c > 0$, so the discriminant is positive and there are two different real solutions.
0 points: Anything less, including a response that is only partly correct.

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DOK 4 · Extended thinking

Why DOK 4: Students carry out an extended investigation, testing cases, generalizing with algebra, and analyzing a new condition before synthesizing a conclusion, which is extended thinking.

A student conjectures: "For a fixed amount of fencing, the rectangle with the greatest area is always a square."

Investigate this conjecture.

- With 40 feet of fencing around all four sides, make a table of at least four possible widths and their areas. Then write the area A as a function of the width w , rewrite it by completing the square, and use the result to justify which rectangle has the greatest area.
- Generalize: show that for any total length of fencing P around all four sides, the greatest area occurs when the rectangle is a square.
- Now suppose one side of the rectangle is a barn wall, so the P feet of fencing are used for only the other three sides (two widths and one length). Write the area function, find the dimensions that give the greatest area, and decide whether the conjecture still holds.

(d) Write a final statement that tells when the conjecture is true and when it is not, supported by your work.

Model response: (a) The length is $20 - w$, so $A(w) = w(20 - w) = -w^2 + 20w = -(w - 10)^2 + 100$. Sample table: $w = 4, A = 64$; $w = 6, A = 84$; $w = 8, A = 96$; $w = 10, A = 100$; $w = 12, A = 96$. Since $-(w - 10)^2 \leq 0$, the greatest area is 100 square feet when $w = 10$ and the length is 10: a square.

(b) With P feet of fencing, the length is $\frac{P}{2} - w$, so $A(w) = w\left(\frac{P}{2} - w\right) = -\left(w - \frac{P}{4}\right)^2 + \frac{P^2}{16}$. The maximum occurs at $w = \frac{P}{4}$, and then the length is $\frac{P}{2} - \frac{P}{4} = \frac{P}{4}$. The best rectangle is always a square.

(c) With fencing on two widths and one length, the length is $P - 2w$ and $A(w) = w(P - 2w) = -2\left(w - \frac{P}{4}\right)^2 + \frac{P^2}{8}$. The maximum occurs at $w = \frac{P}{4}$, so the length is $P - \frac{P}{2} = \frac{P}{2}$, twice the width. The best rectangle is not a square, so the conjecture fails here.

(d) The conjecture is true when the fencing goes around all four sides. It is false when one side is a wall; then the greatest area occurs when the side parallel to the wall is twice as long as each side perpendicular to it.

Rubric: 1 point: Three of the four parts complete and correct.

0 points: Anything less, including a response that is only partly correct.

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3C State logical assumptions being used

Assessed at:

DOK 1

DOK 2

DOK 3

DOK 4

DOK 2 • Skills & concepts

Why DOK 2: Students identify the hidden assumption that time is positive and explain why taking a square root requires it, connecting a step to its condition.

A basketball player's vertical jump height can be modeled using the equation $h = \frac{1}{2}gt^2$, where h is height in feet, g is the gravitational constant, and t is time in seconds. A coach claims that if a player can jump to height h , then the time spent in the air is $t = \sqrt{\frac{2h}{g}}$. The coach derives this by rewriting $h = \frac{1}{2}gt^2$ as $t^2 = \frac{2h}{g}$ and then taking the square root of both sides to get $t = \sqrt{\frac{2h}{g}}$.

Part A: Which logical assumption must the coach be making when deriving the time equation from $t^2 = \frac{2h}{g}$?

Choose one answer.

- ☐ That the height h must be greater than or equal to 32 feet
- ☐ That the basketball player's mass does not affect the jump height
- ☐ That the gravitational constant g equals 32 feet per second squared
- ☒ That t is positive (representing time after the jump begins) ✓

Part B: Which statement best explains why this assumption is necessary for the coach's derivation?

Choose one answer.

- ☒ Because when taking the square root of t^2 , we get $|t| = \sqrt{\frac{2h}{g}}$, which gives both positive and negative solutions; the coach must assume $t > 0$ to select the physically meaningful solution for time ✓
- ☐ Because rational exponents like $\frac{1}{2}$ require all variables to be positive numbers
- ☐ Because $\sqrt{\frac{2h}{g}}$ can only be evaluated when g is a positive constant



Because the original equation $h = \frac{1}{2}gt^2$ only applies to basketball players, not other athletes

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Test Bank question

DOK 3 · Strategic thinking

Why DOK 3: Students must uncover unstated assumptions behind a model and reason about how changing each one would affect the conclusions, which calls for strategic thinking beyond computing.

A coach models the height of a basketball shot, in feet, t seconds after it is released with $h(t) = -16t^2 + 32t + 6$. The coach concludes: "The ball reaches a maximum height of 22 feet after 1 second and would hit the floor after about 2.17 seconds."

- (a) Verify the coach's two numerical conclusions.
- (b) Identify at least three assumptions the coach must be making for the model and these conclusions to be valid.
- (c) For two of your assumptions, explain how the coach's conclusions would change if that assumption were not true.

Model response: (a) The vertex is at $t = -\frac{32}{2(-16)} = 1$, and $h(1) = -16 + 32 + 6 = 22$ feet. Setting $h(t) = 0$ gives $8t^2 - 16t - 3 = 0$, so $t = \frac{16 \pm \sqrt{352}}{16}$; the positive root is $t \approx 2.17$ seconds (the negative root is rejected). Both conclusions are correct.

(b) Possible assumptions: the ball is released at $t = 0$ from a height of 6 feet; only gravity acts on the ball, so air resistance is ignored; nothing such as the rim, the backboard, or another player interrupts the ball's path; the floor is level at height 0; the model applies only for $0 \leq t \leq 2.17$, since negative times and heights below the floor have no meaning.

(c) Sample: If air resistance is not negligible, the ball would rise to less than 22 feet and reach the floor sooner than 2.17 seconds. If the ball hits the rim, the model stops describing the ball at that moment, so the 2.17-second landing time would not happen. If the ball were released from higher than 6 feet, the maximum height and the landing time would both be greater.

Rubric: 1 point: Verifies both numerical conclusions, states at least three reasonable assumptions, and correctly explains how two of the conclusions would change if an assumption were false.

0 points: Anything less, including a response that is only partly correct.

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3D Use the technique of breaking an argument into cases

Assessed at: DOK 1 **DOK 2** DOK 3 DOK 4**DOK 2 • Skills & concepts****Why DOK 2:** Students compute the discriminant and use the three discriminant cases to decide how many solutions exist and what that means in context.

Use the drop-down menus to complete the reasoning about when a quadratic equation has real solutions.

A soccer ball's height above ground after a kick is modeled by $h(t) = -16t^2 + 32t + 6$, where t is time in seconds. To determine if the ball ever reaches a height of 22 feet, we set $-16t^2 + 32t + 6 = 22$ and solve $-16t^2 + 32t - 16 = 0$. Using case analysis on the discriminant $\Delta = b^2 - 4ac$: The discriminant equals 0 ▾. Since Δ equals zero ▾, the quadratic equation has one ▾ real solutions, which means the ball will ▾ reach exactly 22 feet.

Drop-down 1 choices: -512 **0 ✓** 256Drop-down 2 choices: is positive is negative **equals zero ✓**Drop-down 3 choices: **has one ✓** has no has twoDrop-down 4 choices: may or may not will not **will ✓**

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Test Bank question

DOK 3 · Strategic thinking

Why DOK 3: Students must set up exhaustive cases (odd n ; even n with nonnegative or negative x), analyze each, and argue why the cases cover every possibility.

A sports statistician models a runner's change in speed with a real number x (positive means speeding up, negative means slowing down). To simplify formulas, she uses the identity $(x^n)^{1/n} = x$, where n is a positive integer, and claims this is true for every real number x and every positive integer n . Break this claim into the necessary cases based on whether n is odd or even (and, when needed, the sign of x), determine whether the claim holds in each case, and explain why your set of cases covers every possibility. Use an example runner speed such as $x = -2$ to support your reasoning.

Model response: There are three cases, and together they cover every possible combination of n and x :

Case 1 (n odd, any real x): Since odd roots preserve sign, $(x^n)^{1/n} = x$ always. Example: $((-2)^3)^{1/3} = (-8)^{1/3} = -2$. The claim holds.

Case 2 (n even, $x \geq 0$): $x^n \geq 0$, and taking the positive n th root gives back x , so $(x^n)^{1/n} = x$. The claim holds.

Case 3 (n even, $x < 0$): $x^n > 0$, but the n th root symbol/rational exponent gives the nonnegative root, so $(x^n)^{1/n} = |x| = -x \neq x$. Example: $x = -2$, $n = 2$: $((-2)^2)^{1/2} = 4^{1/2} = 2 \neq -2$. The claim fails.

These three cases are exhaustive because every integer n is either odd or even, and for even n every real x is either nonnegative or negative — no other possibilities exist. Since Case 3 produces a counterexample, the statistician's general claim $(x^n)^{1/n} = x$ is false for negative speed changes when n is even; it must instead be written as $|x|$ in that case.

Rubric: 1 point: Correctly separates the argument into the three exhaustive cases (odd n ; even n with $x \geq 0$; even n with $x < 0$), correctly analyzes each case using rational exponent reasoning, gives a valid counterexample for the failing case, and explicitly explains why the cases are exhaustive.

0 points: Anything less, including a response that is only partly correct.

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Test Bank question

3E Distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in the argument—explain what it is

Assessed at: DOK 1 DOK 2 DOK 3 DOK 4 DOK 4 is assessed only in the performance task.

DOK 2 • Skills & concepts

Why DOK 2: Students check each step of a worked solution and recognize that a negative number of games does not make sense in context, locating the flaw.

A basketball analyst models the point differential of a team after x games using the expression $x^2 - 64$. She wants to find the number of games after which the differential equals zero. Select the step where her reasoning first contains an error.

Step 1: Start with the expression $x^2 - 64$, representing the point differential after x games.

Step 2: Recognize the expression as a difference of squares and rewrite it as $(x - 8)(x + 8)$.

Step 3: Set the expression equal to zero: $(x - 8)(x + 8) = 0$, so $x - 8 = 0$ or $x + 8 = 0$.

Step 4: Solve to get $x = 8$ or $x = -8$, and conclude both values represent valid numbers of games played. ✓

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Test Bank question

DOK 3 · Strategic thinking

Why DOK 3: Students must analyze several arguments, including a subtle overgeneralization that fails for values between 0 and 1, and decide which reasoning is valid.

A coach is analyzing athletic performance using exponential models. For each statement about rewriting expressions with rational exponents, determine whether the reasoning is correct or contains a logical flaw.

	Correct Reasoning	Flawed Reasoning
To rewrite $\sqrt[4]{x^3}$ using rational exponents, we apply the rule $\sqrt[n]{a^m} = a^{m/n}$, so $\sqrt[4]{x^3} = x^{3/4}$. This works because the denominator of the rational exponent matches the index of the radical.	☒	☐
Since $5^{1/2} = \sqrt{5}$ and $5^{1/3} = \sqrt[3]{5}$, it follows that $5^{1/6}$ must equal $\sqrt{5} \cdot \sqrt[3]{5}$ because we can multiply the expressions together.	☐	☒
When simplifying $(16^{1/2})^{1/2}$, we can use the power rule $(a^m)^n = a^{mn}$ to get $16^{(1/2)(1/2)} = 16^{1/4}$, which equals 2 because $16^{1/4} = \sqrt[4]{16}$. This method works regardless of the order in which we apply the operations.	☒	☐
To compare $\sqrt[3]{x^2}$ and $\sqrt{x}$ for $x > 0$, we convert both to rational exponents: $x^{2/3}$ and $x^{1/2}$. Since $2/3 > 1/2$, we can conclude that $x^{2/3} > x^{1/2}$ for all positive values of x .	☐	☒
The expression $32^{2/5}$ can be rewritten as $(32^{1/5})^2$, which equals $2^2 = 4$, because $32 = 2^5$ means $32^{1/5} = 2$. We could also compute this as $(32^2)^{1/5} = 1024^{1/5} = 4$, and both methods are valid because the power rule preserves equality.	☒	☐

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Test Bank question

DOK 4 · Extended thinking

Why DOK 4: Students investigate many cases, diagnose a flawed generalization, then build and prove a corrected rule, an extended analysis that goes beyond a single error check.

Riley studied quadratic functions written in vertex form, $y = a(x - h)^2 + k$, and wrote this rule:

"The number of x -intercepts depends only on k . If $k < 0$, the graph has two x -intercepts. If $k = 0$, it

has one. If $k > 0$, it has none."

(a) Test Riley's rule with at least four functions that use different combinations of signs for a and k . For each function, find the actual number of x -intercepts and record whether Riley's rule gives the right answer.

(b) Explain the flaw in Riley's reasoning.

(c) Write a corrected rule that works for every quadratic function in vertex form, and justify it algebraically by solving $a(x - h)^2 + k = 0$ for $(x - h)^2$.

(d) Explain why Riley's rule gives the right answer for every function with $a > 0$.

Model response: (a) Sample tests:

- $y = (x - 1)^2 - 4$: $a > 0, k < 0$; intercepts at $x = -1$ and $x = 3$ (two). Rule correct.

- $y = (x - 1)^2 + 4$: $a > 0, k > 0$; no intercepts. Rule correct.

- $y = -(x - 1)^2 + 4$: $a < 0, k > 0$; intercepts at $x = -1$ and $x = 3$ (two). Rule says none, so it is wrong.

- $y = -(x - 1)^2 - 4$: $a < 0, k < 0$; no intercepts. Rule says two, so it is wrong.

- $y = -2(x + 3)^2$: $k = 0$; one intercept at $x = -3$. Rule correct.

(b) Riley looked only at k , the height of the vertex. Whether the graph reaches the x -axis also depends on whether the parabola opens up or down, which is given by the sign of a .

(c) Solving $a(x - h)^2 + k = 0$ gives $(x - h)^2 = -\frac{k}{a}$. A square cannot be negative. If $-\frac{k}{a} > 0$ (a and k have opposite signs), there are two x -intercepts, $x = h \pm \sqrt{-\frac{k}{a}}$. If $k = 0$, there is one, $x = h$. If $-\frac{k}{a} < 0$ (a and k have the same sign), there are none.

(d) When $a > 0$, $-\frac{k}{a}$ has the opposite sign of k , so "opposite signs" is the same as " $k < 0$." Riley's rule matches the correct rule for every parabola that opens upward, which is why it seemed true.

Rubric: 1 point: Three of the four parts complete and correct.

0 points: Anything less, including a response that is only partly correct.

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3F Base arguments on concrete referents such as objects, drawings, diagrams, and actions

Assessed at: DOK 1 **DOK 2** DOK 3 DOK 4

DOK 2 • Skills & concepts

Why DOK 2: Students connect a cube model to the meaning of a one-third exponent and choose the reasoning that matches it, linking a concrete picture to a rule.

A baker models how baking time relates to dough size using $t = 5 \cdot d^{1/3}$, where d is dough weight and t is baking time in minutes. The baker explains that $d^{1/3}$ means "the number that, when multiplied by itself three times, equals d ." She then says $2^{1/3}$ represents the cube root of 2 because $(2^{1/3})^3 = 2$. Which of the following best completes the baker's reasoning using this concrete model?

- ☐ Therefore, $2^{1/3} \approx 1.26$, and multiplying 1.26 by itself three times gives roughly 2.
- ☐ Therefore, $2^{1/3} = \frac{2}{3}$, since the denominator tells us to divide the base by 3.
- ☒ Therefore, $2^{1/3}$ equals the side length of a cube with volume 2, showing that $(2^{1/3})^3 = 2^{3/3} = 2^1 = 2$. ✓
- ☐ Therefore, $d^{1/3}$ is always smaller than d , so baking time will always be less than the dough weight.

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Test Bank question

A coach has a storage crate shaped like a cube used to hold sports balls. The crate has a volume of 64 cubic feet, and it is built from small unit cubes (1 ft x 1 ft x 1 ft) stacked evenly in a 4-by-4-by-4 arrangement. (a) Use this concrete cube arrangement to explain why $64^{1/3} = 4$. (b) The coach then looks at just one face of the crate, a 4-by-4 square of unit squares, and uses it to represent $64^{2/3}$. Explain, using this square and the properties of exponents, why $64^{2/3} = 16$, and explain how the cube and square models together show that $64^{2/3} = (64^{1/3})^2$.

Model response: Since the crate is built from 4 layers of 4-by-4 unit cubes, it contains $4 \times 4 \times 4 = 64$ unit cubes, so a cube with edge length 4 has volume 64; this means the number whose cube is 64 (i.e., $64^{1/3}$) is 4, matching the definition of a cube root. The face of the crate is a 4-by-4 square made of 16 unit squares, so its area is $4^2 = 16$. Since $64^{1/3} = 4$ from the cube model, squaring that result gives $(64^{1/3})^2 = 4^2 = 16$, which matches the face's area. By the properties of integer exponents, $(64^{1/3})^2 = 64^{2/3}$, so the cube model (finding the edge length) and the square model (finding the face area) together physically demonstrate that $64^{2/3} = 16$: the cube gives the base value 4, and the square shows what happens when that value is raised to the second power, confirming the rational exponent rule $a^{m/n} = (a^{1/n})^m$.

Rubric: 1 point: Correctly identifies edge length 4 from the cube model, correctly computes the square's area as 16, and clearly explains how both concrete models together justify $64^{2/3} = (64^{1/3})^2 = 16$, connecting the physical arrangement to the exponent rule.

0 points: Anything less, including a response that is only partly correct.

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Test Bank question

3G Determine conditions under which an argument does and does not apply

Assessed at: DOK 1 DOK 2 DOK 3 DOK 4

DOK 2 • Skills & concepts

Why DOK 2: Students test a general claim on specific values and identify the case (negative base, even root) where it does not apply.

At a science fair, Maya is presenting a poster about rational exponents. She claims: 'For any real number a and positive integer n , the expression $a^{1/n}$ always equals $\sqrt[n]{a}$ and is a real number.' Use the table to test her claim on different values of a and n , and complete the missing entries.

a	n	$a^{1/n}$	Is the result a real number?
32	5	2	yes
-32	5	-2	yes
-16	2	not real	no ▼
81	4	3	yes

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A track analyst is modeling a sprinter's stride data and writes the rule $(x^2)^{1/2} = x$ to simplify expressions where x represents a sprinter's velocity change (in meters per second), which can be positive (speeding up) or negative (slowing down). The analyst claims this rule, following from the properties of exponents, works for every real number x . Determine the conditions on x for which the analyst's rule is valid and the conditions for which it fails. If it fails, state the correct general rule that applies for all real x , and explain why the exponent property that works for integer exponents needs modification when extended to rational exponents like $1/2$.

Model response: The rule $(x^2)^{1/2} = x$ only holds when $x \geq 0$. Since $x^{1/2}$ denotes the principal (non-negative) square root, $(x^2)^{1/2}$ must equal a non-negative number. For $x \geq 0$, $(x^2)^{1/2} = x$ since x itself is non-negative and squares back to x^2 . But if $x < 0$, say $x = -3$ (a sprinter slowing down by 3 m/s), then $x^2 = 9$ and $9^{1/2} = 3$, not -3 . So the correct general rule for all real x is $(x^2)^{1/2} = |x|$. This modification is necessary because when extending integer exponent rules (where $(x^n)^{1/n}$ simply undoes raising to the n th power for any real x when n is odd) to even denominators, the rational exponent $1/2$ is defined only to produce a non-negative output, so the sign information about x is lost when x is squared first. Thus the analyst's rule applies only to non-negative velocity changes; for negative changes, the absolute value form must be used instead.

Rubric: 1 point: complete and correct with clear reasoning.

0 points: Anything less, including a response that is only partly correct.

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CLAIM 4

Claim 4 · Modeling & Data Analysis

Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems. In the Test Bank, Claims 2, 3 and 4 are built in the performance task (PT) step: choose PT, then Area 2 · Claims 2 + 4. Each question card is marked with its claim and target (for example "Claim 4 · Tgt A"), so generate a set at the DOK you want and keep the cards for the target you need.

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4A Apply mathematics to solve problems from everyday life (Modeling)

Assessed at: DOK 1 **DOK 2** DOK 3 DOK 4**DOK 2 • Skills & concepts**

Why DOK 2: Students apply a given model to a real situation and solve by isolating the cube root and cubing, a short two-step process.

Sports scientists modeling cycling performance found that a cyclist's top speed v (in meters per second) relates to their power output P (in watts) by the equation $v = 4P^{1/3}$. If a cyclist reaches a top speed of 12 meters per second, find the power output P , in watts, using this model.

Correct answer: **27**

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Test Bank question

DOK 3 · Strategic thinking

Why DOK 3: Students must build a profit model, find its maximum, recognize that a constraint rules it out, and reason about where the best allowed price is, which requires planning and justification.

The student council will sell T-shirts. A survey suggests that if the price is p dollars, they will sell $300 - 10p$ shirts. Each shirt costs the council 8 dollars. The supplier requires a minimum order of 150 shirts, and the council will order exactly as many shirts as it expects to sell.

What price should the council charge to earn the greatest possible profit while meeting the supplier's minimum order? Show your work, and explain why the price that maximizes the profit function by itself is not the answer.

Model response: Profit is (price minus cost) times the number sold: $P(p) = (p - 8)(300 - 10p) = -10p^2 + 380p - 2400$. The vertex is at $p = -\frac{380}{2(-10)} = 19$, giving a profit of $(11)(110) = 1210$ dollars. However, at 19 dollars the council expects to sell only $300 - 190 = 110$ shirts, which is below the minimum order of 150. The minimum order requires $300 - 10p \geq 150$, so $p \leq 15$. Because the parabola opens downward with its vertex at $p = 19$, profit increases as p increases toward 19, so on the allowed prices $p \leq 15$ the greatest profit is at $p = 15$: $(15 - 8)(150) = 1050$ dollars. The council should charge 15 dollars per shirt.

Rubric: 1 point: Writes a correct profit model, finds the unconstrained maximum at 19 dollars, recognizes that it violates the minimum order, determines that $p \leq 15$, and justifies that 15 dollars (profit 1050 dollars) is the best allowed price.

0 points: Anything less, including a response that is only partly correct.

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4B Construct chains of reasoning to justify mathematical models

Assessed at: DOK 1 DOK 2 DOK 3 DOK 4 DOK 4 is assessed only in the performance task.

DOK 2 • Skills & concepts

Why DOK 2: Students choose the explanation that connects the square-root shape to diminishing returns, interpreting a model's form at a basic level.

A strength coach models the relationship between an athlete's training intensity and muscle growth. After analyzing data, the coach proposes that muscle fiber thickness t (in micrometers) can be expressed as $t = 50 \cdot I^{1/2}$, where I is a training intensity index from 0 to 100. The coach justifies this model by noting that: (1) doubling the intensity does not double the growth, (2) increases in thickness become more gradual at higher intensities, and (3) the model uses a rational exponent because muscle adaptation follows a square-root pattern rather than a linear or exponential pattern. Which statement best explains why the rational exponent $1/2$ (rather than 1 or 2) is the most appropriate choice for this modeling context?

- ☒ A rational exponent of $1/2$ is appropriate because $50 \cdot I^{1/2} = 50\sqrt{I}$ captures diminishing returns—each unit increase in intensity produces progressively smaller gains in thickness, matching observed biological data where adaptation rates decrease at high intensity. ✓
- ☐ A rational exponent of $1/2$ should be used because it always produces smaller numbers than a linear model, and muscle thickness measurements are typically small values in micrometers.
- ☐ A rational exponent of $1/2$ works best because it converts the intensity index into a square root, which is the standard form for all sports science models regardless of the specific phenomenon being measured.
- ☐ A rational exponent of $1/2$ is better than an exponent of 2 because $I^{1/2}$ and I^2 are reciprocals, so $1/2$ automatically gives the correct inverse relationship for biological systems.

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Test Bank question

DOK 3 · Strategic thinking

Why DOK 3: Students must test three competing models against data, argue which fits and why, and reason about what the exponent implies for larger inputs.

A sports scientist studies how a weightlifter's maximum lift capacity L (in hundreds of kg) depends on body mass m (in kg). She collects three data points: $(m, L) = (8, 4), (27, 9), (64, 16)$. She proposes the power model $L = m^{2/3}$, but a colleague suggests instead the linear model $L = 0.5m$ or the model $L = m^{1/3}$. Rewrite $m^{2/3}$ as a radical expression, use it to test all three models against the data, and explain which model best fits the data and why. Discuss what the exponent implies about how lift capacity scales as body mass increases, and explain the risk of using the linear model to predict lift capacity for a much heavier athlete.

Model response: Since $m^{2/3} = (m^{1/3})^2 = (\sqrt[3]{m})^2$, testing the data: for $m = 8$, $\sqrt[3]{8} = 2$, so $2^2 = 4$ ✓; for $m = 27$, $\sqrt[3]{27} = 3$, so $3^2 = 9$ ✓; for $m = 64$, $\sqrt[3]{64} = 4$, so $4^2 = 16$ ✓. The model $L = m^{2/3}$ fits all three points exactly. The linear model $L = 0.5m$ only matches the first point ($0.5(8) = 4$) but fails badly for the others ($0.5(27) = 13.5 \neq 9$; $0.5(64) = 32 \neq 16$), overestimating lift capacity as mass grows — a serious risk if used to predict values for heavier athletes, since it assumes lift capacity grows proportionally with mass rather than sublinearly. The model $L = m^{1/3}$ fails even at the first point ($8^{1/3} = 2 \neq 4$), so it is rejected outright. Because $L = m^{2/3}$ matches every data point exactly while the alternatives fail, it is the best choice; the exponent $2/3$ reflects a sublinear (diminishing-returns) relationship, consistent with strength scaling roughly with muscle cross-sectional area rather than total body volume, meaning heavier athletes gain lift capacity more slowly than their mass increases.

Rubric: 1 point: Correctly rewrites $m^{2/3}$ as $(\sqrt[3]{m})^2$, verifies it fits all three data points exactly, shows the linear and $m^{1/3}$ models fail with specific calculations, identifies $m^{2/3}$ as the best model, and explains the sublinear scaling implication and risk of the linear model for larger masses.

0 points: Anything less, including a response that is only partly correct.

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Test Bank question

DOK 4 · Extended thinking

Why DOK 4: Students build a model from data, justify its form, optimize under a whole-number condition, then revise for costs and judge limits, an extended chain of modeling reasoning.

The soccer team surveyed families about a car wash fundraiser. The table shows the price per car and the number of cars the team expects at that price.

Price per car (dollars)	4	6	8	10	12
Expected cars	130	110	90	70	50

(a) Show that a linear function fits the expected number of cars, and write it as a function of the price p .

- (b) Use your function to write a revenue model $R(p)$. Explain why revenue is modeled by a quadratic function even though the number of cars is linear.
- (c) Find the price that gives the greatest revenue. The team will charge only a whole-dollar price. Which price or prices should they choose? Justify your answer.
- (d) Supplies cost 1.50 dollars per car. Write a profit model and decide whether the best whole-dollar price changes. Justify your decision with calculations.
- (e) Describe one limitation of using this model to predict the number of cars at a price of 18 dollars.

Model response: (a) Each 2-dollar increase in price lowers the expected number of cars by 20, a constant rate of -10 cars per dollar, so the data are linear. Using $(4, 130)$: $c(p) = 130 - 10(p - 4) = 170 - 10p$. Check: $c(12) = 50$.

(b) $R(p) = p \cdot c(p) = p(170 - 10p) = -10p^2 + 170p$. Revenue is a price times a number of cars, and the number of cars itself depends on price, so the product contains a p^2 term. Raising the price earns more per car but loses cars, so revenue rises and then falls, which is a quadratic pattern.

(c) The vertex is at $p = -\frac{170}{2(-10)} = 8.5$, with $R(8.5) = 722.50$ dollars. For whole-dollar prices, $R(8) = 8 \cdot 90 = 720$ and $R(9) = 9 \cdot 80 = 720$. These are equally good and better than any other whole-dollar price, so the team can choose 8 or 9 dollars.

(d) Profit = $(p - 1.5)(170 - 10p) = -10p^2 + 185p - 255$, with vertex at $p = 9.25$. Profit at 9 dollars: $7.5 \cdot 80 = 600$ dollars. Profit at 10 dollars: $8.5 \cdot 70 = 595$ dollars. Profit at 8 dollars: $6.5 \cdot 90 = 585$ dollars. The best whole-dollar price is now 9 dollars, so the supply cost breaks the tie.

(e) At 18 dollars the model predicts $170 - 180 = -10$ cars, which is impossible. The model only makes sense for prices up to 17 dollars, and the survey only covered prices from 4 to 12 dollars, so predictions far outside that range are unreliable.

Rubric: 1 point: Four parts correct and justified.

0 points: Anything less, including a response that is only partly correct.

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4C State logical assumptions being used

Assessed at: **DOK 1** **DOK 2** **DOK 3** **DOK 4****DOK 1 • Recall**

Why DOK 1: Students recognize a standard modeling assumption, that air resistance is ignored, which is identifying a familiar fact rather than reasoning about it.

A coach models the distance a shot put travels using the formula $d = v_0^{2/3}$, where d is the distance in meters and v_0 is the initial velocity. Which of the following is a logical assumption the coach is making in this model?

- ☐ The shot put will travel in a perfectly straight line regardless of angle.
- ☒ The effects of air resistance and other real-world factors are negligible and can be ignored. ✓
- ☐ The initial velocity must be expressed as a fraction rather than a whole number.
- ☐ The distance traveled is inversely proportional to the square root of velocity.

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Test Bank question

Why DOK 2: Students must connect the scaling factor to the shape of the crate and explain which assumption makes the rational-exponent model valid.

A strength coach stores weight plates in cube-shaped crates. For a cube with side length s feet, the volume is $V = s^3$, so solving for the side length gives $s = V^{1/3}$. The coach wants to build a new crate with twice the volume of his current crate, so he calculates the new side length as $s_{new} = s_{old} \cdot 2^{1/3}$. Which assumption must hold for this rational-exponent model to correctly give the new side length?

- ☒ The crate must remain a cube, with all three dimensions scaling by the same factor as volume changes ✓
- ☐ The weight of material used to build the crate stays exactly the same when volume doubles
- ☐ The formula $s = V^{1/3}$ only applies when the side length s is a whole number
- ☐ The volume of the crate increases at a constant rate as the side length increases

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Test Bank question

DOK 3 · Strategic thinking

Why DOK 3: Students analyze several stated assumptions about domain, structure, and population, judging which hold and which are unjustified, which requires evaluating the model's limits.

A coach models the relationship between athletes' training intensity (measured on a scale from 1 to 10) and their vertical jump height using the equation $h = 18 + 6\sqrt[3]{t}$, where h is jump height in inches and t is training intensity. For each statement about the assumptions embedded in this model, determine whether it is true or false.

	True	False
The model assumes that the relationship between training intensity and jump height follows a cube root function, which implicitly assumes that increases in training intensity produce diminishing returns in jump improvement—a key structural assumption needed to justify why the rate of height gain slows as t increases.	☒	☐
The model assumes that training intensity values outside the stated 1–10 scale (such as $t = 0$ or $t = 15$) will produce reliable predictions, meaning no boundary conditions or practical limitations constrain the model's domain.	☐	☒
The model assumes that all athletes respond identically to the same training intensity level, which means individual variation in factors like genetics, prior training history, and biomechanics are either negligible or averaged across the athlete population.	☒	☐
The model assumes that jump height depends only on current training intensity and is independent of cumulative fatigue, recovery time, or changes in training methods—an assumption that simplifies reality but may break down over longer time periods.	☒	☐
The constant term 18 in the model represents the baseline jump height when training intensity is zero, which is a valid assumption for all athletes regardless of their initial fitness level.	☐	☒

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Test Bank question

4D Interpret results in the context of a situation

Assessed at: DOK 1 **DOK 2** DOK 3 DOK 4**DOK 2 • Skills & concepts**

Why DOK 2: Students check a computed result against real-world speeds and interpret whether it makes sense, connecting the math to context.

A sports analyst is building a model to estimate race times. She proposes the formula $T = 0.6d^{1/2}$, where T is the predicted time in seconds and d is the race distance in meters. She uses the formula to predict the time for a 400-meter race: $T = 0.6(400)^{1/2} = 0.6(20) = 12$ seconds. Which statement best interprets whether this result makes sense?

☐ The result makes sense because 12 seconds is less than the current world record of about 43 seconds for 400 meters, so the model gives a reasonable, achievable time.

☒ The result does not make sense because a time of 12 seconds for 400 meters implies an average speed of about 33 m/s, which is far faster than any human can run (top sprinters reach only about 10 m/s), so the model is unrealistic at this distance. ✓

☐ The result does not make sense because $400^{1/2}$ should have been calculated as $400/2 = 200$, not 20, so the time should be much larger.

☐ The result makes sense because the units of the formula work out correctly, so any value the formula produces must be physically reasonable.

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DOK 3 · Strategic thinking

Why DOK 3: Students compute two outputs, compare the ratio to a manager's claim, and justify with exponent reasoning why area grows more slowly than volume.

A sporting goods company models the amount of rubber coating needed for a training ball using $C = 4.8V^{2/3}$, where C is the coating area in square inches and V is the ball's volume in cubic inches. A standard ball has $V = 36$ cubic inches, and a larger training ball has $V = 288$ cubic inches (8 times the volume). The manager assumes that because the volume is 8 times larger, the company should budget 8 times as much rubber coating for the larger ball. Calculate the coating area C for each ball, and use the properties of rational exponents to explain whether the manager's reasoning makes sense. Should the coating budget scale by a factor of 8? Justify your answer.

Model response: For the standard ball: $C = 4.8(36)^{2/3} \approx 4.8(10.90) \approx 52.3$ square inches. For the larger ball: $C = 4.8(288)^{2/3} \approx 4.8(43.6) \approx 209.3$ square inches. The ratio of coating areas is $209.3/52.3 \approx 4$, not 8. This makes sense because $288/36 = 8$, and $8^{2/3} = (2^3)^{2/3} = 2^2 = 4$: since volume scales with the cube of the linear dimension but surface area (and coating) scales with the square, an 8-fold increase in volume corresponds to only a 4-fold increase in coating area. The manager's assumption is incorrect — the company should budget about 4 times as much coating, not 8, since surface area grows more slowly than volume as size increases.

Rubric: 1 point: Correctly computes both coating areas (about 52.3 and 209.3 square inches), identifies the ratio as 4 (not 8), and explains using rational-exponent reasoning ($8^{2/3} = 4$) why area scales more slowly than volume, connecting this to the manager's flawed budgeting assumption.

0 points: Anything less, including a response that is only partly correct.

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Test Bank question

4E Analyze adequacy and make improvements to existing models

Assessed at: DOK 4 DOK 2 DOK 3 DOK 4 DOK 4 is assessed only in the performance task.

DOK 2 • Skills & concepts

Why DOK 2: Students compare a model's predictions to data and interpret the mismatch to choose a better model, which takes more than one step but follows a direct path.

A coach recorded the height of a tossed ball at several times.

Time t (seconds)	0	0.5	1	1.5	2
Height (feet)	5	17	21	17	5

An assistant used the first two data points to build the linear model $h = 24t + 5$. Which statement best evaluates the linear model and gives a better model?

- ☐ The linear model is adequate because it matches the heights at $t = 0$ and $t = 0.5$.
- ☒ The linear model is not adequate because it predicts the ball keeps rising (29 feet at $t = 1$ and 53 feet at $t = 2$) while the data rise and then fall; the quadratic model $h = -16t^2 + 32t + 5$ matches every data point. ✓
- ☐ The linear model is not adequate, but the steeper linear model $h = 32t + 5$ would match every data point.
- ☐ The linear model is not adequate; the quadratic model $h = 16t^2 + 32t + 5$ matches every data point.

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DOK 3 · Strategic thinking

Why DOK 3: Students analyze residuals to judge a model's adequacy, test an improved model, and defend which prediction to trust, which requires reasoning from evidence across several steps.

A driving instructor collected data on the total stopping distance of a car at different speeds.

Speed v (mph)	20	30	40	50	60
Stopping distance d (feet)	40	75	120	175	240

A student proposes the linear model $d = 5v - 60$.

- (a) Find the residual (actual minus predicted) for each data point.
- (b) Describe the pattern in the residuals and explain what it shows about the linear model.
- (c) Test the model $d = 0.05v^2 + v$ against the data, and explain whether it is an improvement.
- (d) Use both models to predict the stopping distance at 70 mph. Which prediction would you trust more for a safety poster? Explain why.

Model response: (a) Predictions from $d = 5v - 60$: 40, 90, 140, 190, 240. Residuals: $40 - 40 = 0$, $75 - 90 = -15$, $120 - 140 = -20$, $175 - 190 = -15$, $240 - 240 = 0$.

(b) The residuals are 0, negative in the middle, and 0 again, forming a U-shaped (curved) pattern instead of a random scatter around 0. This shows the data curve upward and a straight line is the wrong shape: it overestimates in the middle and will underestimate beyond the data.

(c) $d = 0.05v^2 + v$ gives $20 + 20 = 40$, $45 + 30 = 75$, $80 + 40 = 120$, $125 + 50 = 175$, and $180 + 60 = 240$. Every residual is 0, so the quadratic model is a clear improvement.

(d) Linear: $5(70) - 60 = 290$ feet. Quadratic: $0.05(4900) + 70 = 315$ feet. I would trust the quadratic prediction, because it fits all the data and follows the curved pattern. The linear model underestimates stopping distance at higher speeds, which would be dangerous on a safety poster.

Rubric: 1 point: Computes all residuals correctly, explains that the curved residual pattern shows the linear model is the wrong shape, verifies that the quadratic model fits the data, and makes both predictions at 70 mph with a reasoned choice of the quadratic model.

0 points: Anything less, including a response that is only partly correct.

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DOK 4 · Extended thinking

Why DOK 4: Students analyze residuals, revise a model, apply it under new conditions, and evaluate its limits, an extended investigation across several conditions.

A physics class launches a ball straight up from the top of a 6-foot platform. A textbook model for the height of a ball launched upward at 20 feet per second is $h(t) = -16t^2 + 20t$, where h is the height in feet and t is the time in seconds. The class recorded this data:

t (seconds)	0	0.25	0.5	0.75	1
Height above the ground (feet)	6	10	12	12	10

- (a) Compute the textbook model's prediction and the residual (actual minus predicted) for each time. Describe the pattern.
- (b) Explain what the pattern reveals about why the textbook model is not adequate here, and write an improved model.
- (c) Use both models to predict when the ball hits the ground. Explain which prediction is reasonable and why.
- (d) The class plans a second trial with the same launch speed from a 10-foot platform. Adjust your model and predict when the ball will hit the ground, to the nearest hundredth of a second.
- (e) Describe one factor not included in any of these models that could make real results differ, and explain how it could affect the results.

Model response: (a) Textbook predictions $h(t) = -16t^2 + 20t$: 0, 4, 6, 6, 4. Residuals: 6, 6, 6, 6, 6. Every residual is exactly 6 feet.

(b) A constant residual of 6 means the textbook model has the right shape but is shifted down by 6 feet: it assumes the ball starts at ground level, but it was launched from a 6-foot platform. Improved model: $h(t) = -16t^2 + 20t + 6$, which matches all five data points.

(c) Textbook model: $-16t^2 + 20t = 0$ gives $t = 0$ or $t = 1.25$ seconds. Improved model: $-16t^2 + 20t + 6 = 0$ gives $8t^2 - 10t - 3 = 0$, so $(4t + 1)(2t - 3) = 0$ and $t = 1.5$ seconds (the root $t = -0.25$ is rejected). The improved prediction of 1.5 seconds is reasonable because it accounts for the extra 6 feet the ball must fall; the textbook model predicts landing on the platform level, not the ground.

(d) From a 10-foot platform: $h(t) = -16t^2 + 20t + 10$. Setting $h(t) = 0$ gives $8t^2 - 10t - 5 = 0$, so $t = \frac{10 + \sqrt{260}}{16} \approx 1.63$ seconds.

(e) Sample: Air resistance is ignored. It would slow the ball, so the ball would not rise as high as predicted and the landing time would differ from the model's value. Another factor: if the actual launch speed is not exactly 20 feet per second, a faster launch would make the ball land later and a slower launch would make it land sooner.

Rubric: 1 point: Four parts correct.

0 points: Anything less, including a response that is only partly correct.

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4F Identify important quantities in a practical situation and map their relationships (e.g., using diagrams, two-way tables, graphs, flow charts, or formulas)

Assessed at: **DOK 1** **DOK 2** **DOK 3** **DOK 4**

DOK 1 • Recall

Why DOK 1: Students combine exponents with one rule and name the input and output quantities, a direct identification task.

Use the drop-down menus to complete the statement.

A track coach models an athlete's "training load index" using the expression $L = 4t^{1/2} \cdot t^{1/3}$, where t is the number of minutes the athlete trains. Applying the properties of exponents, this expression can be rewritten as $L = 4t^{5/6}$. In this model, the independent (input) quantity is the training time t , and the dependent (output) quantity that the coach is trying to determine is the training load index L .

Drop-down 1 choices: $4t^{2/5}$ $8t^{5/6}$ $4t^{1/6}$ $4t^{5/6}$ ✓

Drop-down 2 choices: the number of athletes the training load index L the training time t ✓

Drop-down 3 choices: the training load index L ✓ the training time t the constant 4

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A sports analyst builds a model for a 100-meter training program. For a sprint of distance d meters, she models the completion time as $t(d) = 2d^{1/2}$ seconds, and defines average speed as $v(d) = \frac{d}{t(d)}$ meters per second. Which statements correctly describe the quantities and relationships in this system? Select all that apply.

- ☒ In this system, distance d is the independent variable, while time t and speed v are dependent variables that respond to changes in d . ✓
- ☒ Because $d^{1/2} = \sqrt{d}$, increasing the distance from 4 meters to 16 meters changes the time from $t(4) = 2\sqrt{4} = 4$ seconds to $t(16) = 2\sqrt{16} = 8$ seconds. ✓
- ☒ Substituting $t(d) = 2d^{1/2}$ into $v(d) = \frac{d}{t(d)}$ gives $v(d) = \frac{1}{2}d^{1/2}$, showing that v and t are linked through the same rational exponent $\frac{1}{2}$ rather than by independent formulas. ✓
- ☐ Time t is the independent variable in this model, and it determines both d and v .
- ☐ Since $d^{1/2}$ means d divided by 2, tripling the distance d will triple the time t .
- ☐ The exponent $\frac{1}{2}$ in $d^{1/2}$ means the distance d is squared before the time is calculated.
- ☒ If the distance d decreases, the model predicts that the time t also decreases, since t is an increasing function of d . ✓

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Test Bank question

DOK 3 · Strategic thinking

Why DOK 3: Students must identify several related quantities, map them onto a right-triangle diagram, apply two different relationships, and decide which constraint controls, which requires strategic thinking.

A drone operator stands in an open field. The drone flies at a constant altitude of 150 feet. Two safety rules apply:

- The straight-line distance from the operator to the drone must be at most 400 feet.
- The angle of elevation from the operator to the drone must be at least 30° so the drone stays visible above a tree line.

- (a) Draw and label a diagram that shows the operator, the drone, the altitude, the horizontal distance d , the line-of-sight distance s , and the angle of elevation θ .
- (b) Write an equation or inequality that relates d to each rule.
- (c) Determine the greatest horizontal distance the drone can fly from the operator while following both rules, to the nearest tenth of a foot. Explain which rule limits the distance and why.

Model response: (a) The diagram is a right triangle: the operator at one acute angle, the drone at the top, the 150-foot altitude as the vertical leg, the horizontal distance d as the horizontal leg, the line of sight s as the hypotenuse, and the angle of elevation θ at the operator.

(b) Rule 1: $s = \sqrt{d^2 + 150^2} \leq 400$, so $d^2 + 22500 \leq 160000$. Rule 2: $\tan \theta = \frac{150}{d}$, and $\theta \geq 30^\circ$ means $\frac{150}{d} \geq \tan 30^\circ$, so $d \leq \frac{150}{\tan 30^\circ}$.

(c) Rule 1 gives $d \leq \sqrt{137500} \approx 370.8$ feet. Rule 2 gives $d \leq \frac{150}{\tan 30^\circ} = 150\sqrt{3} \approx 259.8$ feet. Both rules must hold, so the greatest horizontal distance is about 259.8 feet. The angle rule is the one that limits the distance: at 259.8 feet the line of sight is only $\sqrt{259.8^2 + 150^2} = 300$ feet, well under 400, but any farther and the angle of elevation drops below 30° .

Rubric: 1 point: Draws a correctly labeled right-triangle diagram, writes a correct relationship for each rule, finds both limits (about 370.8 ft and 259.8 ft), and concludes that the angle rule limits the distance to about 259.8 ft with an explanation.

0 points: Anything less, including a response that is only partly correct.

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4G Identify, analyze, and synthesize relevant external resources to pose or solve problems

Assessed at: DOK-1 DOK-2 **DOK 3** DOK 4 Assessed only in the performance task, at DOK 3 and 4.

DOK 3 • Strategic thinking

Why DOK 3: Students must judge which of two external sources is reliable and why, then use that information in the model, combining evaluation of evidence with computation.

A sports-biomechanics researcher models a sprinter's peak ground reaction force F (in newtons) as a function of the sprinter's maximum velocity v (in meters per second) using the equation $F = 200v^{3/2}$. Two resources report the sprinter's measured maximum velocity: (1) a peer-reviewed biomechanics journal, using laser-timing gates calibrated to 0.01 s accuracy, reports $v = 9$ m/s; (2) a fitness-tracking phone app, using consumer GPS known to overestimate speed due to signal lag, reports $v = 9.61$ m/s. Using the more reliable data source, find F , in newtons.

Correct answer: **5400**

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Test Bank question

DOK 4 • Extended thinking

Why DOK 4: Students must select relevant information from several sources, ignore distractors, build and analyze a model, and synthesize a recommendation, which is extended thinking.

The softball coach wants to use a pitching machine indoors to launch balls straight up for pop-fly practice. She has gathered these resources:

Resource 1 (physics reference): An object launched straight up with an initial velocity of v_0 feet per second from an initial height of h_0 feet has height $h(t) = -16t^2 + v_0t + h_0$ feet after t seconds, if air resistance is ignored.

Resource 2 (machine manual): The machine launches balls from a height of 3 feet. Its speed dial can be set anywhere from 30 to 60 feet per second. The coach's usual setting is 48 feet per second.

Resource 3 (gym facility sheet): The ceiling is 40 feet high. Light fixtures hang 4 feet below the ceiling. The gym floor is 94 feet by 50 feet.

Resource 4 (weather report): Outdoor winds today are 15 miles per hour from the west.

(a) Decide which resources and which pieces of information are relevant to whether a ball will hit the lights, and explain why the others are not needed.

- (b) Determine whether a ball launched at the usual setting will hit the light fixtures. Show your work.
- (c) Find the greatest whole-number speed setting, in feet per second, that keeps the ball at least 1 foot below the light fixtures.
- (d) Write a short recommendation to the coach that uses your results and states one assumption that could affect it.

Model response: (a) Relevant: Resource 1 (the height formula), the 3-foot launch height and the speed settings from Resource 2, and the ceiling height and light position from Resource 3 (lights hang at $40 - 4 = 36$ feet). Not needed: the floor dimensions (the ball goes straight up) and Resource 4 (outdoor wind does not affect an indoor launch).

(b) At the usual setting, $h(t) = -16t^2 + 48t + 3$. The vertex is at $t = \frac{48}{32} = 1.5$ seconds, and $h(1.5) = -36 + 72 + 3 = 39$ feet. Since 39 feet is above the 36-foot lights, the ball will hit the light fixtures.

(c) The maximum height for launch speed v_0 occurs at $t = \frac{v_0}{32}$, giving $h_{max} = 3 + \frac{v_0^2}{64}$. To stay at least 1 foot below the lights: $3 + \frac{v_0^2}{64} \leq 35$, so $v_0^2 \leq 2048$ and $v_0 \leq 45.25$. The greatest whole-number setting is 45 feet per second (maximum height about 34.6 feet). A setting of 46 gives about 36.1 feet, which is too high.

(d) Sample: "Do not use the usual 48 ft/s setting indoors; the balls would reach about 39 feet and hit the lights at 36 feet. Set the machine to 45 ft/s or lower, which keeps the highest point about 34.6 feet. This assumes the machine's speed dial is accurate and that the launch height stays at 3 feet; if the dial reads low, choose a lower setting for safety."

Rubric: 1 point: Three of the four parts correct.

0 points: Anything less, including a response that is only partly correct.

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Sources

- Smarter Balanced Assessment Consortium, 2015–16 Summative Technical Report: claim statements, and the Appendix 2.A test blueprints (pp. 43–48), which list the DOK levels assessed for every claim and target. <https://www.cde.ca.gov/ta/tg/ca/documents/sb16sbtechrpt.pdf>
- Target wording and standards: Smarter Balanced Mathematics Content Specifications and the CCSS for Mathematics, as stored in the AchieveMetrics Test Bank.
- DOK level definitions: Smarter Balanced Content Specifications (Webb Depth of Knowledge), summarised on the "How to use" page.
- Example questions: the AchieveMetrics Test Bank, reviewed for this guide.

